

## Exam: Algebra

Hamburg University of Applied Science  
 Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering  
 Prof. Robert Heß, Jan 26<sup>th</sup> 2012, duration: 90 Min.

Result: ..... of 68 points

Mark: ..... points.

**Problem 1 (12 points)**

Prove by complete induction: 
$$\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

**Problem 2 (10 points)**

For the boolean set  $B = \{0, 1\}$  express the function  $f : B^3 \rightarrow B, (a, b, c) \mapsto (a \wedge b) \vee c$  in conjunctive normal form.

**Problem 3 (6 points)**

List the three solution behaviours of a system of linear equations.

**Problem 4 (16 points)**

Investigate the following system of linear equations:

$$a - 2b - c = 1 \qquad 2a + b - c = 6 \qquad 2a - b + 2c = 1 \qquad a - 4c = 6$$

1. Evaluate the rank of the coefficient matrix.
2. Evaluate the rank of the extended coefficient matrix.
3. Derive from the ranks the solution behaviour and explain your conclusion.
4. Solve the system of linear equations.

**Problem 5 (16 points)**

Let the matrix  $A = \begin{pmatrix} 0 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$  be a linear map  $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3, x \mapsto Ax$ .

1. Evaluate the dimension of the domain of  $L$
2. Evaluate the dimension of the image of  $L$
3. Evaluate the dimension of the kernel of  $L$
4. For a system of linear equations  $Ax = b$  with  $A$  as given above and  $b \in \text{image}(L)$  what is the solution behaviour? Explain why.

**Problem 6 (8 points)**

Let  $A$  be a square matrix with determinant  $\det(A) = 5$ . The square matrix  $B$  has been derived from matrix  $A$  by the following operations: a) transpose of  $A$ , b) multiplication of row 2 by 4, c) exchange row 2 with row 3 and d) multiplication of row 1 by  $-\frac{1}{5}$ .

What is the determinant of  $B$ ?