Surname: ..... Forename: ..... MatrNo.: .....

Exam: Calculus 1

Hamburg University of Applied Science
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering
Prof. Robert Heß, Jan 30th 2012, duration: 90 Min.

Result: ..... of 100 points Mark: ..... points.

Problem 1 (12 points)

Resolve/simplify the following complex expressions:

$$a = \mathrm{j}^{10} \qquad b = \mathrm{Im}\left((3+4\mathrm{j})^2\right) \qquad c = \arg(1-\mathrm{j}) \qquad d = z \cdot \overline{z} - |z|^2, z \in \mathbb{C}$$

Problem 2 (14 points)

Find the radius of convergence and sketch the region of convergence on the complex plane:

$$f : \mathbb{C} \to \mathbb{C}, z \mapsto \sum_{k=0}^{\infty} (2z)^k$$

Problem 3 (12 points)

Which of the following statements are true?

1. Every continuous function is differentiable.
2. A continuous function $f : \mathbb{R} \to \mathbb{R}$ with $f(1) < 0$, $f(2) > 0$ has exactly one zero on the interval $(1, 2)$.
3. Every differentiable function is continuous.
4. If a series $\sum_{k=0}^{\infty} a_n$ is convergent, then the sequence (a_n) is a zero sequence.

Problem 4 (18 points)

Differentiate the following expressions:

$$a = \frac{\mathrm{d}}{\mathrm{d}x}\sqrt{\sin(2\pi x)} \qquad b = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2 - x - 1}{x^2 - 9} \qquad c = \frac{\mathrm{d}^n}{\mathrm{d}t^n}\hat{p}e^{\mathrm{j}(2\pi f t + \varphi)}$$

Problem 5 (24 points)

Separate the following function into partial fractions: $f(x) = \dfrac{x^2 - 4x + 5}{x^3 - 6x^2 + 11x - 6}$

Problem 6 (20 points)

Analyse the polynomial $p_4 = \begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto x^4 + 2x^3 - 12x^2 + 14x - 5 \end{cases}$ with respect to inflection points.