

## Exam: Algebra

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Prof. Robert Heß, July 2<sup>nd</sup> 2012, duration: 90 Min.

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Result: ..... of 68 points                      Mark: ..... points.

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**Question 1 (12 points)**

Prove by mathematical induction that  $4n^3 - n$ ,  $n \in \mathbb{N}$  has the common divisor 3.

**Question 2 (8 points)**

Express the following terms by conjunction and negation only as short as possible:

1.  $x_1 \vee x_2$                       2.  $\overline{a \vee b \vee c}$                       3.  $x \rightarrow y$                       4.  $u \leftrightarrow v$

**Question 3 (16 points)**

Given is the following system of linear equations:

$$x_1 + 2x_2 - x_3 = 2 \quad x_1 + 2x_2 - 2x_3 = 3 \quad x_1 + x_2 + x_3 = -1 \quad 2x_1 - 3x_3 = 1$$

1. Solve the SLE by Gauss-Jordan elimination.
2. Are the equations linear dependent? Explain your answer.
3. Does the SLE contain a contradiction? Explain your answer.

**Question 4 (8 points)**

A system of linear equations with four unknowns has a coefficient and extended coefficient matrix with rank three.

1. Does the SLE show a contradiction? Explain why.
2. What kind of solution do you expect? Explain why.

**Question 5 (16 points)**

Let the matrix  $A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 3 & 2 \end{pmatrix}$  be a linear map  $L : \mathbb{R}^4 \rightarrow \mathbb{R}^4, x \mapsto Ax$ .

1. Evaluate the dimension of the domain of  $L$
2. Evaluate the dimension of the image of  $L$
3. Evaluate the dimension of the kernel of  $L$
4. For a system of linear equations  $Ax = b$  with  $A$  as given above and  $b \in \text{image}(L)$  what is the solution behaviour? Explain why.

**Question 6 (8 points)**

Evaluate the volume of the parallelepiped spanned by the three vectors  $v_1 = (-1, 2, 1)^T$ ,  $v_2 = (1, -1, 2)^T$  and  $v_3 = (0, 2, 3)^T$ .