

Exam: Linear Algebra

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Prof. Dr. Robert Heß, January 29th 2013, duration: 90 Min.

Result: of 100 points Mark: points.

Problem 1 (20 points)

Prove by mathematical induction: $n^2 + n$ is even

Problem 2 (12 points)

List a) the three elementary row operations that do not influence the value of the unknowns of a system of linear equations and b) their influence on the determinant of the coefficient matrix.

Problem 3 (20 points)

Solve the following system of linear equations by Gauss-Jordan elimination:

$$x + 2y + 3z = -2 \quad 3x + 4y + 2z = 1 \quad 3x + y + z = 2 \quad 2x + 2y + 4z = -2$$

Problem 4 (12 points)

Let A be the coefficient matrix and $A|b$ the extended coefficient matrix of a system of five linear equations with four unknowns. Answer the following questions and explain your answers.

- a) What is the maximum rank of A ?
- b) What is the maximum rank of $A|b$?
- c) Can the rows of A be linear independent?

Problem 5 (22 points)

For $A = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 3 \\ 2 & 3 & 4 & 4 \\ 2 & 2 & 2 & 3 \end{pmatrix}$ evaluate A^{-1} and $\det(A)$.

Problem 6 (14 points)

A parallelepiped has one corner at the origin of a coordinate system and its three neighbour corners at the coordinates $(-1,0,0)$, $(3,2,0)$ and $(1,2,3)$. What is the volume of the parallelepiped?