

## Exam: Algebra

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Prof. Dr. Robert Heß, 27.1.2014, duration: 90 min.

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Result: ..... of 100 points                      Mark: ..... points.

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### **Problem 1 (18 points)**

Prove by mathematical induction:  $n^3 - n$  has the divisor 6 for all  $n \in \mathbb{N}$

### **Problem 2 (16 points)**

List a) the three elementary row operations that do not influence the value of the unknowns of a system of linear equations and b) their influence on the determinant of the coefficient matrix.

### **Problem 3 (15 points)**

A system of linear equations has five unknowns and seven equations. a) What is the maximum rank of the coefficient matrix? b) What is the maximum rank of the extended coefficient matrix? Explain your Answers!

### **Problem 4 (18 points)**

$$x + 2y + z = 1 \qquad x + y + z = 1 \qquad 3x + 2y + 2z = 2 \qquad 2x + 2y + z = 1$$

For the given system of linear equations evaluate the ranks of coefficient matrix and extended coefficient matrix and draw your conclusion on the solution behaviour.

### **Problem 5 (18 points)**

For  $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 2 & 2 & 1 \end{pmatrix}$  find  $A^{-1}$  and  $\det(A)$ .

What is the volume of a parallelepiped spanned by the three column vectors of  $A$ ?

### **Problem 6 (15 points)**

Let  $A$  be a matrix with determinant  $\det(A) = 4$ . Matrix  $B$  has been derived from matrix  $A$  by the following operations:

- a) transpose  $A$ ,
- b) exchange row 1 with row 3,
- c) multiply row 1 by  $-\frac{1}{4}$ ,
- d) subtract row 1 from row 2 and
- e) multiply row 2 by 3.

What is the determinant of  $B$ ?