

## Exam: Algebra

Hamburg University of Applied Science  
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering  
Prof. Dr. Robert Heß, 30.6.2014, duration: 90 Min.

Result: ..... of 100 points                      Mark: ..... points.

### Problem 1 (20 points)

Prove by mathematical induction:  $1 + \sum_{k=1}^n \frac{2^{2k-2}}{3^k} = \left(\frac{4}{3}\right)^n$

### Problem 2 (10 points)

What is the difference in solution behaviours for homogeneous und inhomogeneous SLEs?

### Problem 3 (25 points)

A) Convert the following extended coefficient matrix into its reduced row echelon form.

$$\left( \begin{array}{ccc|c} -1 & -2 & 3 & 0 \\ 1 & 1 & -2 & -1 \\ 2 & -1 & 2 & -2 \\ 3 & 2 & 1 & -3 \end{array} \right)$$

- B) What is the rank of the coefficient matrix?  
C) What is the rank of the extended coefficient matrix?  
D) What does this mean for the solution behaviour?

### Problem 4 (20 points)

Solve the following SLE:

$$a + b + c + d = 2, \quad b - c + d = 1, \quad -a + c + d = -2, \quad a - b = -1$$

### Problem 5 (15 points)

The spanning vectors of a parallelepiped are arranged as column vectors of a  $3 \times 3$  matrix which has been converted into an identity matrix by the following operations: a) once swapping two rows, b) multiplying a row by 3 and another row by  $\frac{1}{2}$  and c) a few times adding multiple of rows to other rows. What is the volume of the parallelepiped?

### Problem 6 (10 points)

What is the rank of the matrix  $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$  ?