Exam: Mathematics 1

Hamburg University of Applied Science Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering Prof. Dr. Robert Heß, 18.1.2017, duration: 90 Min. Permitted aids: up to six A4-pages of personal notes (i.e. single sided sheets)

Result: of 100 points Mark: points.

Problem 1 (12 points)

A functions codomain and image are equal. What can you say about injectivity, surjectivity and bijectivity for this function? Explain your answer.

Problem 2 (12 points)

Find all solutions in Cartesian form for $z \in \mathbb{C}$ with $z^4 = -64$.

Problem 3 (14 points)

Analyse convergence by ratio test and sketch the region of convergence for:

$$f(z) = \sum_{k=0}^{\infty} \frac{(z+2-j)^k}{2^k}, z \in \mathbb{C}$$

Problem 4 (12 points)

With $\omega \in \mathbb{R}$ and $n \in \mathbb{N}$ resolve and simplify the following expressions:

$$a = \frac{\mathrm{d}}{\mathrm{d}x} \tan(\sin(x))$$
 $b = \frac{\mathrm{d}^{4n}}{\mathrm{d}x^{4n}} \sin(\omega x)$ $c = \frac{\mathrm{d}}{\mathrm{d}x} \frac{\arctan(x)}{x^2 + 1}$

Problem 5 (20 points)

Apply partial fraction decomposition on: $\frac{4x^2 - x + 1}{x^3 - 3x^2 + 7x - 5}$

Problem 6 (30 points) a) Evaluate the inverse of $A = \begin{pmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 \\ 3 & 3 & 3 & 2 \end{pmatrix}$.

- b) Derive the determinant of A.
- c) What is the maximum rank of a matrix $M(3 \times 4, \mathbb{R})$? Explain your answer.
- d) What is the determinant of a matrix $M(3 \times 4, \mathbb{R})$? Explain your answer.