Surname: .... Forename: .... MatrNo.: ....

Exam: Mathematics 1

Hamburg University of Applied Science
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering
Prof. Dr. Robert Heß, February 16th 2021, duration: 90 Min.
Permitted aids: six A4-pages of personal notes (i.e. single sided sheets), lecture notes and other personal notes

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Result: .... of 100 points Mark: .... points.

Problem 1 (20 points)

Prove by mathematical induction: $\frac{9}{4}+\sum_{k=1}^{n}\frac{3^{k+1}}{2^{k+2}}=\left(\frac{3}{2}\right)^{n+2}$

Problem 2 (15 points)

Analyse convergence by root test and sketch the region of convergence for:

$$f(z)=\sum_{k=0}^{\infty}\left(\frac{z-\mathrm{j}}{2}\right)^k,\quad z\in\mathbb{C}$$

Problem 3 (12 points)

Resolve, i.e. differentiate and simplify the following expressions:

a) $\frac{\mathrm{d}}{\mathrm{d}t}\,e^{\mathrm{j}\omega t+\sigma t}$ b) $\frac{\mathrm{d}}{\mathrm{d}x}\,\frac{x-1}{x+1}$ c) $\frac{\mathrm{d}^n}{\mathrm{d}b^n}\,\exp(ab+bc+ca)$

Problem 4 (24 points)

Apply partial fraction decomposition on the following rational function with separate summands for all poles except for pairs of complex conjugate poles.

$$f(x)=\frac{11x-23}{x^3-2x^2-5x+6}$$

Problem 5 (20 points)

Analyse the following SLE:

$$x_1+2x_2+3x_3=4 \qquad 4x_1+5x_2+6x_3=4 \qquad 2x_1+2x_2=-6 \qquad 3x_2+4x_3=6$$

a) Evaluate the extended coefficient matrix $A|b$

b) Find the reduced row echelon form $\mathrm{rref}(A|b)$

c) Evaluate $\mathrm{rank}(A)$ and $\mathrm{rank}(A|b)$ and draw you conclusion w.r.t. the solution behaviour

d) If possible derive the solution of the SLE

Problem 6 (9 points)

Let $A\in M(5\times 5,\mathbb{K})$ be a non-invertible matrix. What can you say about a) the determinant of A, b) linear dependency of row vectors in A and c) the number of basic columns in A.