Surname: ....   Forename: ....   MatrNo.: ....

<u>Exam: Mathematics 1</u>

Hamburg University of Applied Science
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering
Prof. Dr. Robert Heß, February 18th 2022, duration: 90 Min.
Permitted aids: up to six A4-pages of personal notes (i.e. single sided sheets)

Result: .... of 100 points   Mark: .... points.

I hereby apply to participate in the examination in Mathematics 1 at February 18th 2022.

I feel healthy and fit for the examination. All the preconditions for the examination are fulfilled.

I know that anyone who has symptoms of illness which have have not been clarified by medical staff is not allowed to enter the premises of the HAW Hamburg. I hereby confirm that I do not show any symptoms.

February 18th 2022

....
signature

Problem 1 (16 points)

Prove by mathematical induction: $\displaystyle\sum_{k=1}^{n} \frac{1}{\sqrt{k}} > \sqrt{n}$ for all $n \geq 2, n \in \mathbb{N}$

Problem 2 (15 points)

For $z^3 = 2 - 2\mathrm{j}$ find all solutions for $z \in \mathbb{C}$.

Problem 3 (15 points)

With $f : \mathbb{C} \to \mathbb{C}$ find and sketch the region of convergence for $\displaystyle f(z) = \sum_{k=0}^{\infty} (2z - 1 - \mathrm{j})^k$

Problem 4 (20 points)

Apply partial fraction decomposition on the following rational function with separate summands for all poles except for pairs of complex conjugate poles.

$$f(x) = \frac{3x^2 + 5x - 2}{x^3 - 2x - 4}$$

Problem 5 (9 points)

Differentiate and simplify the following expressions:

a) $\dfrac{\mathrm{d}}{\mathrm{d}t}\, e^{\mathrm{j}(\omega t + \varphi_0)}$ b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\, \ln|x^2 + y^3|$ c) $\dfrac{\mathrm{d}^{4n}}{\mathrm{d}x^{4n}}\, \cos(ax + b)$

Problem 6 (25 points)

a) Evaluate the inverse of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 2 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 0 \end{pmatrix}$.

b) Derive the determinant of A.