

## Mock-Exam: Algebra

Hamburg University of Applied Science  
Faculty of Engineering & Computer Science, Department of Information and Electrical Engineering  
Prof. Robert Heß, Jan 18<sup>th</sup> 2012, duration: 90 Min.

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Result: ..... of 68 points                      Mark: ..... points.

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### **Problem 1 (12 points)**

Prove by complete induction:  $3^n > n^2$  for  $n \in \mathbb{N}$

### **Problem 2 (10 points)**

For the boolean set  $B = \{0, 1\}$  express the function  $f : B^3 \rightarrow B, (a, b, c) \mapsto b \wedge (a \vee c)$  in disjunctive normal form.

### **Problem 3 (6 points)**

List the three elementary row operations that do not influence the values of unknowns of a system of linear equations.

### **Problem 4 (12 points)**

Solve the following system of linear equations by applying Gauss-Jordan elimination.

$$2x_2 + x_3 = 2 \qquad x_1 + x_3 = -2 \qquad 2x_1 - x_2 - x_3 = 0$$

### **Problem 5 (16 points)**

For  $A = \begin{pmatrix} 0 & -1 & 2 & 1 \\ 1 & -1 & 0 & -1 \\ 2 & -1 & 1 & 2 \\ 2 & 1 & -2 & 2 \end{pmatrix}$  evaluate  $A^{-1}$  and  $\det(A)$ .

### **Problem 6 (12 points)**

Find the solution behaviours of SLEs for the statements below:

1. The rank of the coefficient matrix is less than the rank of the extended coefficient matrix.
2. Let  $A$  be the coefficient matrix that maps the vector of unknowns to the vector of constants. The dimension of the kernel of  $A$  is one and the vector of constants is an element of the image of  $A$ .
3. The rank of the coefficient matrix and extended coefficient matrix equal the number of unknowns.
4. The coefficient matrix is invertible.